JEE Advanced : Paper-II (2010)

IMPORTANT INSTRUCTIONS

A. General:

1. This question paper contains 32 pages having 57 questions.
2. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page no. 32) of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any from are not allowed.
5. Log and Antilog tables are given in page numbers 30 and 31 respectively.
6. The answer sheet, a machine-gradable. Objective Response sheet **(ORS)**, is provided separately.
7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.
8. Do not break the seals of the question–paper booklet before instructed to do so by the invigilation.

B. Filling the Right Part of the ORS:

9. The ORS also has a **CODES** printed on its lower and upper parts.
10. Make are the **CODE** on the **ORS** is the same its that on this booklet. If the Codes do not match, ask **for a change of the Booklet**.
11. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes. Do not the boxes write these anywhere else.
12. Darken the appropriate bubbles under each digit of your Registration No. with HB Pencil.

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Mathematics and Physics). Each part consists of four sections.
14. For each question in **Section I:** you will be awarded **5 marks if you darken only the bubble** corresponding to the correct answer and **zero mark** if no bubbles on darkened. In all other cases, **minus two (-2) mark** will be awarded.
15. For each question in **Section II:** you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. No negative marks will be awarded for incorrect answers in this section.
16. For each question in **Section III:** you will be **awarded 3 marks** if you darken **only** the bubble corresponding to the correct answer and **zero marks** if bubbles are darkened. In all other cases, **minus one (-1)** mark will be awarded.
17. For each question in **Section IV**, you will be awarded **2 marks** for each now in which your darkened the **bubbles(s)** corresponding to the correct answer. Thus each question in this section carries a maximum of **8 marks**. There is no negative marks awarded for incorrect answer(s) in this Section.

PART-A: CHEMISTRY

Useful data

Atomic Numbers : B = 5; C = 6; N = 7; O = 8; F = 9 ; Na = 11; Si = 14; P =15; S = 16; Cl = 17; Ti = 22; V = 23; Cr = 24; Ni = 28; Cu = 29; Br = 35; Rh = 45; Sn = 50; Xe = 54; Tl = 81

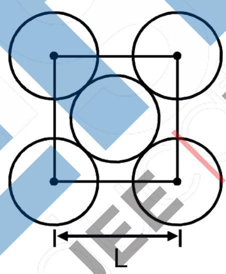
1 amu	=	1.66×10^{-27} kg	e	=	1.6×10^{-19} C
R	=	0.082 L-atm K^{-1} mol^{-1}	c	=	3.0×10^{8} m s^{-1}
h	=	6.626×10^{-34} J s	F	=	96500 C mol^{-1}
N_A	=	6.022×10^{23}	R_H	=	2.18×10^{-18} J
m_e	=	9.1×10^{-31} kg	$4\pi\epsilon_0$	=	1.11×10^{-10} J^{-1} C^2 m^{-1}

SECTION-I

Single Correct Choice Type

This section contains **6 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. The packing efficiency of the two-dimensional square unit cell shown below is

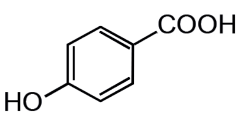


(A) 39.27 % (B) 68.02 % (C) 74.05 % (D*) 78.54 %

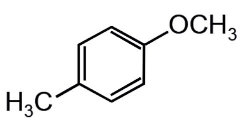
2. Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule B_2 is

(A*) 1 and diamagnetic (B) 0 and diamagnetic
(C) 1 and paramagnetic (D) 0 and paramagnetic

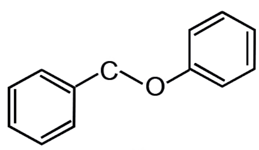
3. The compounds P, Q and S



P

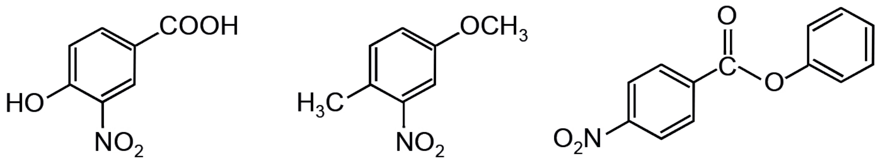


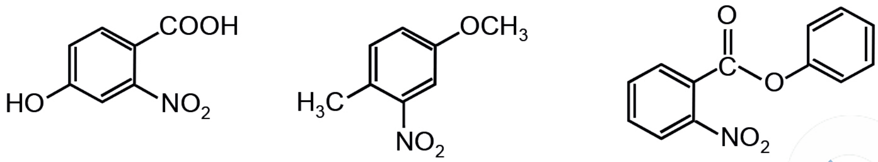
Q

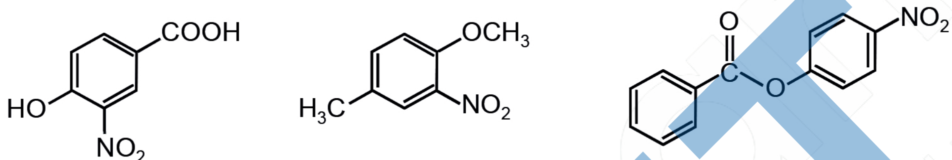


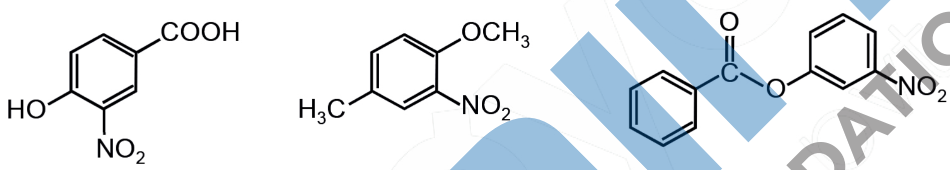
S

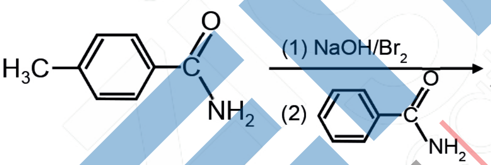
were separately subjected to nitration using HNO_3 / H_2SO_4 mixture. The major product formed in each case respectively, is

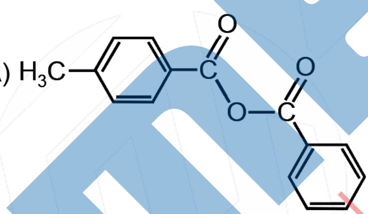
(A) 

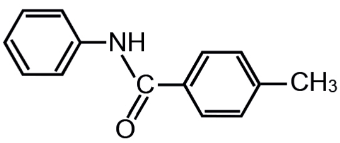
(B) 

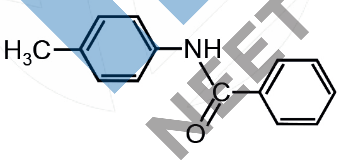
(C*) 

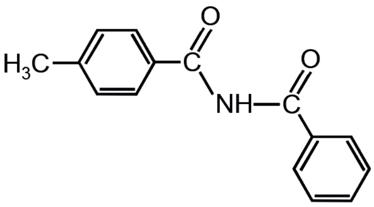
(D) 

4. In the reaction  **T** the structure of the Product T is

(A) 

(B) 

(C*) 

(D) 

5. The complex showing a spin-only magnetic moment of 2.82 B.M.is

(A) $Ni(CO)_4$ (B*) $[NiCl_4]^{2-}$ (C) $Ni(PPh_3)_4$ (D) $[Ni(CN)_4]^{2-}$

6. The species having pyramidal shape is

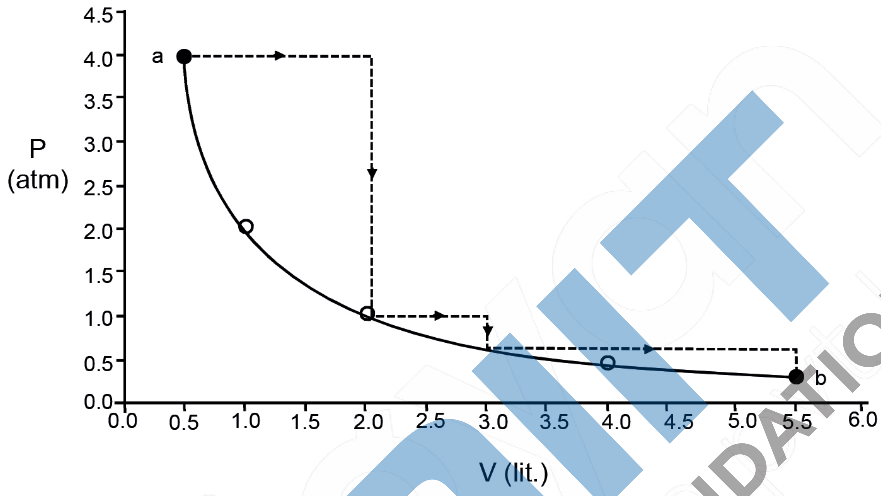
(A) SO_3 (B) BrF_3 (C) SiO_3^{2-} (D*) OSF_2

SECTION-II

(Integer Type)

This Section contains a group of **5 questions**. The answer to each question is a **single digit integer** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

7. One mole of an ideal gas is taken from **a** to **b** along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is w_s and that along the dotted line path is w_d, then the integer closet to the ratio w_d / w_s is



Ans. 2

8. The total number of diprotic acids among the following is

H_3PO_4 H_2SO_4 H_3PO_3 H_2CO_3 $H_2S_2O_7$

H_3BO_3 H_3PO_2 H_2CrO_4 H_2SO_3

Ans. 6

9. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is

Ans. 3

10. Among the following, the number of elements showing only one non-zero oxidation state is

O, Cl, F, N, P, Sn, Tl, Na, Ti

Ans. 2

11. silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 g cm^{-3}. The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as $y \times 10^x$. The value of x is

Ans. 7

SECTION-III

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the first paragraph **3 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 12 to 14

The hydrogen-like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light, the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

12. The state S_1 is

(A) 1s (B*) 2s (C) 2p (D) 3s

13. Energy of the state S_1 in units of the hydrogen atom ground state energy is

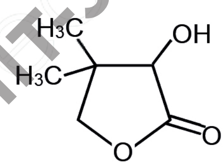
(A) 0.75 (B) 1.50 (C*) 2.25 (D) 4.50

14. The orbital angular momentum quantum number of the state S_2 is

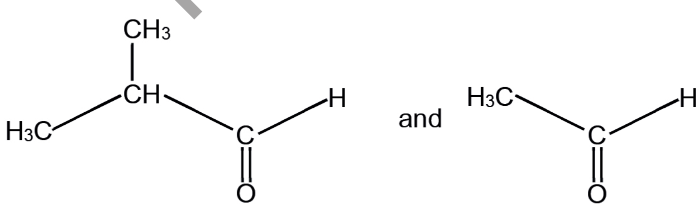
(A) 0 (B*) 1 (C) 2 (D) 3

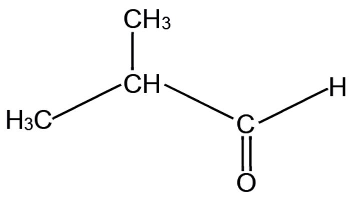
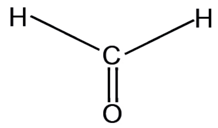
Paragraph for Questions 15 to 17

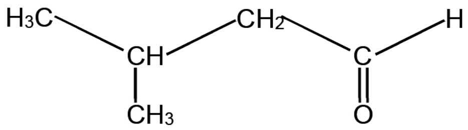
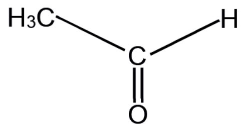
Two aliphatic aldehydes P and Q react in the presence of aqueous K_2CO_3 to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below:

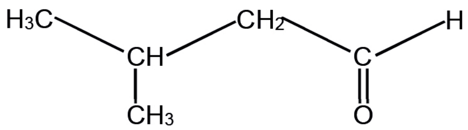
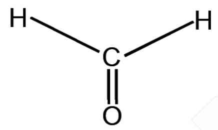


15. The compounds P and Q respectively are

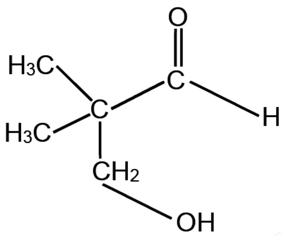
(A) 

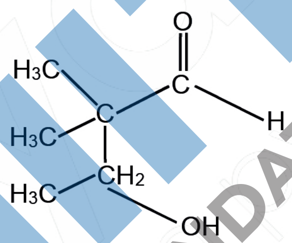
(B)  and 

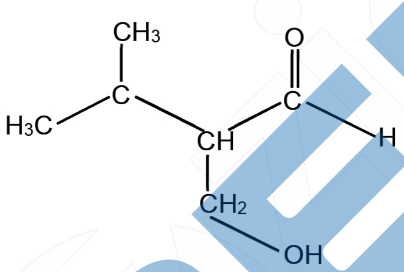
(C)  and 

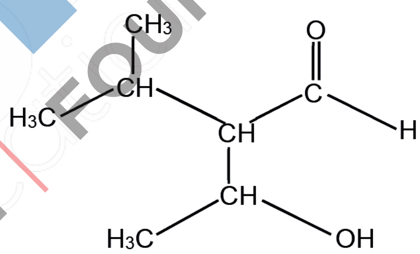
(D)  and 

16. The compound R is

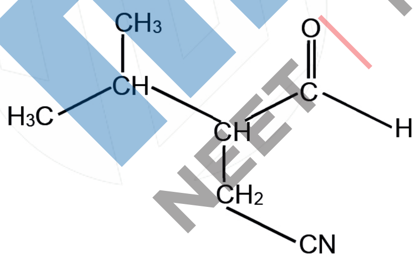
(A) 

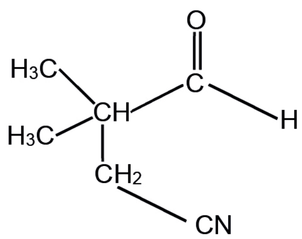
(B) 

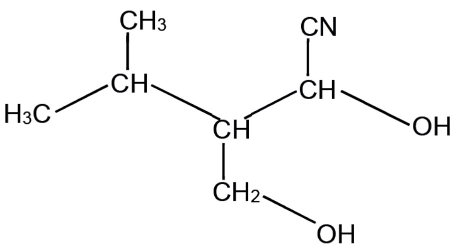
(C) 

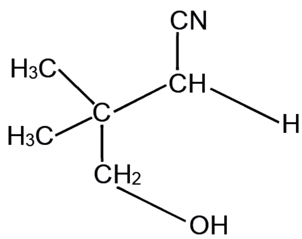
(D) 

17. The compound S is

(A) 

(B) 

(C) 

(D) 

SECTION-IV

(Matric Type)

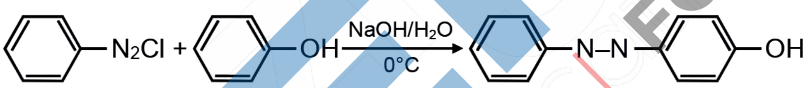
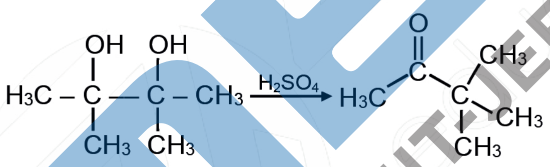
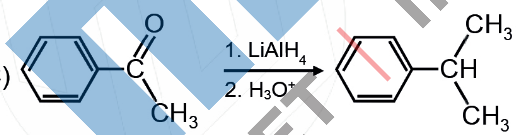
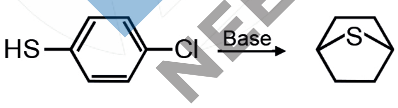
This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

18. All the compounds listed in **Column I** react with water. Match the result of the respective reactions with the appropriate options listed in **Column II**.

	Column I		**Column II**
(A)	$(CH_3)_2SiCl_2$	(p)	Hydrogen halide formation
(B)	XeF_4	(q)	Redox reaction
(C)	Cl_2	(r)	Reacts with glass
(D)	VCl_5	(s)	Polymerization
		(t)	O_2 formation

Ans. (A) p,s (B) p,q,r,t (C) p,q,t (D) p

19. Match the reactions in Column I with appropriate options in **Column II**.

	Column I		**Column II**
(A)		(p)	Racemic mixture
(B)		(q)	Addition reaction
(C)		(r)	Substitution reaction
(D)		(s)	Coupling reaction
		(t)	Carbocation intermediate

Ans. (A) r,s,t (B) t, (C) p, q, (D) r

PART-B: MATHEMATICS

Section-I

(Single Correct Choice Type)

This Section contains **6 multiple choice questions.** Each question has 4 choices A), B). C) and D) out of which ONLY ONE is correct.

20. Two adjacent sides of a parallelogram ABCD are given by

$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

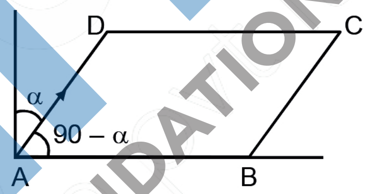
The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Sol. **[B]**

$\cos(90-\alpha) = \frac{-2+20+22}{15\times 3}$ $\Rightarrow$ $\sin\alpha = \frac{8}{9}$

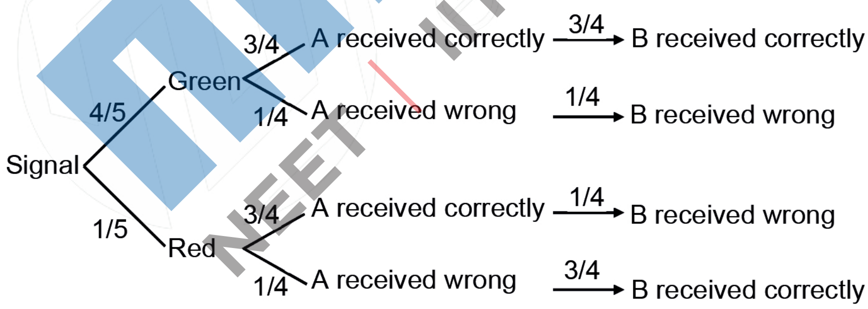
$\Rightarrow$ $\cos\alpha = \frac{\sqrt{17}}{9}$



21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Sol. **[C]**



Probability of B receiving green signal

$$P(G) = \frac{4}{5}\left[\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}\right]+\frac{1}{5}\left[\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}\right]=\frac{1}{2}+\frac{3}{8}=\frac{3}{40}$$

Probability that signal related was green while B received Green = $\frac{\frac{1}{2}}{\frac{1}{2}+\frac{3}{40}} = \frac{20}{23}$ **Ans.]**

22. If the distance of the point P(l, – 2, 1) from the plane x + 2y - 2z = α , where α > 0. is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ (C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

Sol. **[A]**

Perpendicular distance = $\left|\frac{1-4-2-\alpha}{3}\right| = 5$

$\Rightarrow$ $|\alpha + 5| = 15$ $\Rightarrow$ $\alpha = 10$ (as α > 5)

foot of the perpendicular $\frac{x-1}{1}=\frac{y-(-2)}{2}=\frac{z-1}{-2}=\frac{-(1-4-2-10)}{9}$

or $x = \frac{8}{3}$, $y = \frac{4}{3}$ and $z = \frac{-7}{3}$

23. Let f be a real-valued function defined on the interval (–1, 1) such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all $x \in(-1,1)$, and let f^{-1} be the inverse function of f.

Then $(f^{-1})'(2)$ is equal to

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{e}$

Sol. **[B]**

Let $f^{-1}(x) = g(x)$ then $g[f(x)] = x \Rightarrow$ $g'[f(x)]\, f'(x) = 1$

at $x = 0$ $f(x) = 2$ so put $x = 0$ to get

$g'[f(0)]\, f'(0) = 1$

also $f'(x) = e^x\left[2+\int_0^x \sqrt{1+t^4}\,dt\right] + e^x(1 + x^4) \Rightarrow$ $f'(0) = 3$

Hence $g'(2) = \frac{1}{3}$

24. For r = 0, 1, ... 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r\,(B_{10}\,B_r - C_{10}\,A_r)$ is equal to

(A) $B_{10} - C_{10}$ (B) $A_{10}\left(B_{10}^2 - C_{10}\,A_{10}\right)$ (C) 0 (D) $C_{10} - B_{10}$

Sol. **[D]**

$$\sum_{r=1}^{10} {}^{10}C_r\left({}^{20}C_{10}\,{}^{20}C_r - {}^{30}C_{10}\,{}^{10}C_r\right) = \sum_{r=1}^{10} {}^{20}C_{10}\left({}^{10}C_r\,{}^{20}C_{20-r}\right) - {}^{30}C_{10}\sum_{r=1}^{10}\left({}^{10}C_r\right)^2$$

$= {}^{20}C^{10}({}^{10}C^{20} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1)$

$= B_{10\ 10} - B_{10} - B_{10}C_{10} + C_{10} = C_{10} - B_{10}$

25. Let S = {1, 2, 3, 4). The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25 (B) 34 (C) 42 (D) 41

Sol. **[A]**

{1, 2, 3, 4}

{1, 2, 3, 4}

$\{1\} \rightarrow 4$

$\{1, 2\} \rightarrow {}^4C_2 = 6$

$\{1, 2, 3\} \rightarrow {}^4C_1 = 4$

$\{1, 2, 3, 4\} \rightarrow 1$

$A \rightarrow$ One element – 4

$B \rightarrow$ Two element – ${}^4C_2 = 6$

$C \rightarrow$ Three element – ${}^4C_4 = 4$

$D \rightarrow$ Four element – ${}^4C_4 = 1$

$E \rightarrow$ No elements $\rightarrow \phi$

Pairs from (A) = ${}^4C_2 = 6$

Pairs from (B) = $\frac{{}^4C_2}{3} = 3$

Pairs from (B) = $\frac{{}^4C_2}{3} = 3$

Pairs from (C) = 0

Pairs from (D) = 0

Pairs from E + (A, B, C, D) = 0

Pairs from E + (A, B, C, D) = 0

Pairs from (A – B) = 3

4 × 3 = 12

1 × 4 = 4

Total = 25

Section-II

(Integer Type)

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9. The correct digit below the question no. in the ORS is to be bubbled.

26. Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det(adj B) $=10^6$, then [k] is equal to

[**Note :** adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

Sol. $R_2 \to R_2 + R_3$

$$|A| = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 0 & 1+2k & -(2k+1) \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$$

$C_2 \to C_2 + C_3$

$$= \begin{vmatrix} 2k-1 & 4\sqrt{k} & 2\sqrt{k} \\ 0 & 0 & -(2k+1) \\ -2\sqrt{k} & 2k-1 & -1 \end{vmatrix} = (2k+1)\{(2k-1)^2 + 8k\} = (2k+1)\{4k^2 - 4k + 1 = 8k$$

$= (2k + 1)^3$

$|B| = 0$

det(adj A) + det (adj B) = 10^6

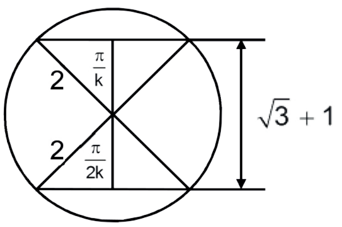
$\left((2k+1)^3\right)^{3-1} + 0 = 10^6 \Rightarrow (2k+1)^6 = 10^6 \Rightarrow 2k + 1 = 10$

$\Rightarrow k = \frac{9}{2} \quad \therefore \quad [x] = 4]$

27. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$ where k > 0, then the value of [k] is

[**Note :** [k] denotes the largest integer less than or equal to k]

Sol.



$x + y = 2\cos 2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3}+1$

$= \cos\frac{\pi}{k} + \cos\frac{\pi}{2k} = \frac{\sqrt{3}+1}{2}$

$= 2\cos^2\frac{\pi}{k} - 1 + \cos\frac{\pi}{2k} = \frac{\sqrt{3}+1}{2}$

$= 2a^2 + a = \frac{\sqrt{3}+3}{2}$

$4a^2 + 2a - \left(\sqrt{3}+3\right) = 0$

$a = \frac{-2 \pm \sqrt{4 + 4\cdot 4\sqrt{3}\left(\sqrt{3}+1\right)}}{8}$

$= \frac{-2 \pm \sqrt{1 + 4\left(3+\sqrt{3}\right)}}{8}$

$= \frac{-1 \pm \sqrt{13 + 4\sqrt{3}}}{4}$

$= \frac{-1 \pm \sqrt{1 + 2\sqrt{3}}}{4}$

$a = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ or $a = \frac{-2-2\sqrt{3}}{4} = -\left(\frac{\sqrt{3}+1}{2}\right)$ (rejected)

$\cos\frac{\pi}{2k} = \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{2k} = \frac{\pi}{6} \Rightarrow k = 3$

28. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Sol. $\frac{1}{2}ab\sin C = 15\sqrt{3}$; $6 \times 10 \times \sin C = 30$

$\sin C = \frac{\sqrt{3}}{2} \Rightarrow C = 120°$ $\{\because$ C is obtuse$\}$

$\cos 120° = \frac{36+100-c^2}{2\times 6\times 10} = \frac{-1}{2} \Rightarrow 136 - c^2 = -60 \Rightarrow c^2 = 196 \Rightarrow c = 14$

$\therefore s = \frac{a+b+c}{2} = \frac{6+10+14}{2} = 15$

$\Rightarrow r = \frac{\Delta}{s} = \frac{15\sqrt{3}}{15} = \sqrt{3} \Rightarrow r^2 = 3$

29. Let f be a function defined on R (the set of all real numbers) such that

$f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$, for all $x \in R$.

If g is a function defined on R with values in the interval $(0, \infty)$ such that

$f(x) = \ln(g(x))$, for all $x \in R$,

then the number of points in R at which g has a local maximum is

Sol. $g(x) = e^{f(x)}$

$g'(cx) = e^{f(x)} f'(x)$

$f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$

$g'(x) = 0 \quad \Rightarrow \quad x = 2009, 2010, 2011, 2012$

for max. $g'(x)$ must change sign from + ve to – ve

x = 2009

+	–	–	+	+
	2009	2010	2011	2012

$\therefore$ 1 **Ans.**

30. Let $a_1, a_2, a_3.....a_{11}$ be real numbers satisfying

$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4..... 11.

If $\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + ... + a_{11}}{11}$ is equal to

Sol. $a_k - a_{k-1} = a_{k-1} - a_{k-1} \Rightarrow$ a_n's are in A.P.

Let common difference d

$$\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = \frac{15^2 + (15+d)^2 + ... + (15+10d)^2}{11} = 990$$

on solving $\Rightarrow$ d = – 3, – 9 (rejected) $a_2 > \frac{27}{2}$

$$\frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{\frac{11}{2}(30 + 10(-3))}{11} = 0$$

SECTION - III (Paragraph Type)

This Section contains **2 paragraphs**. Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices A). B). C) and D) out of which ONLY ONE is correct. **[3, (–1)]**

Paragraph for questions 31 to 33.

Tangents are drawn from the point P(3, 4) to the ellipse touching the ellipse at points A and B.

31. The coordinates of A and B are

(A) (3. 0) and (0. 2)

(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

32. The orthocenter of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{25}\right)$

33. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

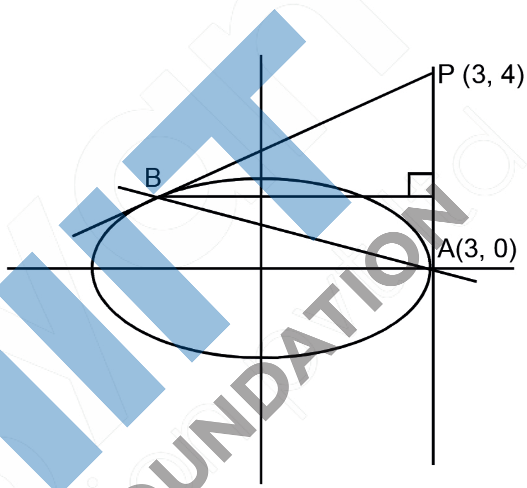
(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Sol.

(31) **[D]**

Clearly A is (3, 0)



$y = mx \pm \sqrt{9m^2 + 4}$

$4 - 3m = \pm\sqrt{9m^2 + 4}$

$16 + 9m^2 - 24m = 9m^2 + 4$

$12 = 24m \quad \Rightarrow \quad m = \frac{1}{2}$

$y = \frac{1}{2}x + \sqrt{\frac{9}{4} + 4}$

$2y = x + 5$

Let Be be (x_1, y_1)

equation of tangent at B

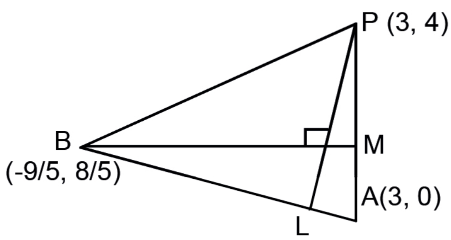
$$\left.\begin{array}{l}\frac{xx_1}{9} + \frac{yy_1}{4} = 1 \\ x - 2y = -5\end{array}\right\} \Rightarrow \frac{x_1}{9 \cdot 1} = \frac{y_1}{4(-2)} = \frac{1}{-5}$$

$x_1 = \frac{-9}{5}, \ y_1 = \frac{8}{5}$

(32) **[C]**

$$m_{AB} = \frac{\frac{8}{5} - 0}{\frac{-9}{5} - 3} = \frac{8}{-24} = \frac{-1}{3}$$

$\therefore$ Altitude P_2 : $y - 4 = 3(x - 3)$



$\Rightarrow \quad 3x - y = 5 \quad ....(1)$

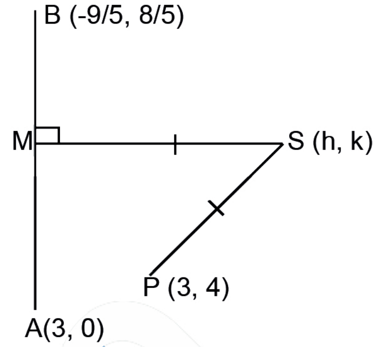
Altitude BM : $y = \frac{8}{5} \quad ....(2)$

Solving (1) and (2): $\left(\frac{11}{5}, \frac{8}{5}\right)$

(33) **[A]**

Straight line AB :

$y - 0 = \frac{-1}{3}(x - 3)$



Applying x + 3y = 3

Applying PS = SM

$(h-3)^2 + (k-4)^2 = \left|\frac{h+3k-3}{\sqrt{10}}\right|^2$

$\therefore$ Locus of S(h, k) is

$10(x^2 + y^2 - 6x - 5y + 25) = x^2 + 9y^2 + 6xy + 9 - 6(x + 3y)$

$9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$]

Paragraph for questions 34 to 36.

Consider the polynomial

$f(x) = l + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of f(x) and let t = | s |.

34. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$ (C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

35. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$ (C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

36. The function f ′(x) is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

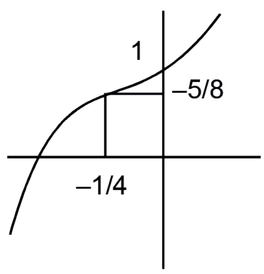
(C) increasing in (– t, t)

(D) decreasing in (– t, t)

Sol.

(34) **[C]**

$f(x) = 1 + 2x + 3x^2 + 4x^3$



$f'(x) = 2 + 6x + 12x^2$

$D = 36 - 4 \cdot 2 \cdot 12 < 0$

$\therefore$ f (x) is strictly increasing

$\Rightarrow$ f (x) = 0 has only one real roots

f (–11) < 0

f (–3/4) > 0

root lies in (– 11, – 3/4)

(35) [A]

Required area = $\int_0^{x_0}\left(1+2x+3x^2+4x^3\right)dx$ t = | s | = x_0

$$= \left(x+x^2+x^3+x^4\right)_0^{x_0} = x_0+x_0+x_0^2+x_0^3+x_0^4 = \left(x_0+x_0^3\right)\left(1+x_0\right)$$

$$= x_0\left(1+x_0^2\right)\left(1+x_0\right) \qquad |x| = x_0 \in \left(\frac{1}{2},\frac{3}{4}\right)$$

$$\therefore \quad \frac{1}{2}\left(1+\frac{1}{4}\right)\left(1+\frac{1}{2}\right) < A < \frac{3}{4}\left(1+\frac{9}{16}\right)\left(1+\frac{3}{4}\right)$$

$$\frac{15}{32} < A < \frac{525}{256} \quad \Rightarrow \quad \textbf{(A)}$$

(36) [B]

f (x) = 1 + 2x + $3x^2$ + $4x^3$

g (x) = f ' (x) = 2 + 6x + $12x^2$

g ' (x) = 6 + 24x = 0 $\Rightarrow$ x = $\frac{-1}{4}$

g (x) is increasing in $\left(\frac{-1}{4},\infty\right)$ and decreasing in $\left(-\infty,\frac{-1}{4}\right)$

t = | s | = x_0 > 1

$\Rightarrow$ increasing in $\left(\frac{-1}{4},t\right)$ and decreasing in $\left(-t,\frac{-1}{4}\right)$

SECTION - IV (Matrix Type)

This Section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. **[2 + 2 + 2 + 2 = 8]**

37. Match the statements in **Column-I** with the values in **Column-II**.

	Column I		Column II
(A)	A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}+\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p)	– 4
(B)	The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q)	0
(C)	Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$ $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$ If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r)	4
(D)	Let f be the function on $[-\pi,\pi]$ given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)/\sin\left(\frac{x}{2}\right)$ for $x\neq 0$	(s)	5
	The value of $\frac{2}{\pi}\int_{-\pi}^{\pi}f(x)dx$ is	(t)	6

Ans. (A) t (B) p,r (C) q (D) r

Sol.

(A) $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}=r$

$\therefore$ P (r + 2– 2 r + 1, r –1)

$\frac{x}{r+2}=\frac{y}{1-2r}=\frac{z}{r-1}=K$

$\frac{K(r+2)-8/3}{2}=\frac{y}{1-2r}=\frac{z}{r-1}=K$

K (r –1 + 1 – 2r) = –2

K (–r) = –2

Kr = 2

$\frac{2+2K=8/3}{2}=\frac{K-2.2+3}{-1}$

$\frac{2K-2/3}{2}=\frac{K-1}{-1}$

$K-\frac{1}{3}+K-1=0$

$\therefore$ K = $\frac{2}{3}$

$\therefore$ r = 3

P (5, –5, 2) Q $\left(5.\frac{2}{3}, -5.\frac{2}{3}, 2.\frac{2}{3}\right)$

d = $\sqrt{5^2+5^2+2^2}.\frac{1}{3}$

$\Rightarrow d^2 = \frac{54}{9} = 6$ Ans.

(B) $\tan^{-1}(x + 3) - \tan^{-1}(x - 3) = \sin^{-1}\left(\frac{3}{5}\right)$

$\frac{6}{1+x^2-9} = \frac{3}{4}$

$8 = x^2 - 8$

$\therefore x = \pm 4$

(C) $\vec{a}.\vec{b} = 0$

$\vec{a} = \mu\vec{b} + \mu\vec{c}$

$\vec{c} = \frac{\vec{a} - \mu\vec{b}}{4}$

$\vec{b} + \vec{c} = b + \frac{\vec{a} - \mu\vec{b}}{4}$

$\vec{b} + \vec{c} = \frac{(4-\mu)\vec{b} + \vec{a}}{4}$

$(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$

$(\vec{b} - \vec{a})\left(\frac{(4-\mu)\vec{b} + \vec{a}}{4}\right) = 0$

$(4 - \mu)\ |\vec{b}|^2 = |\vec{a}|^2$

so $\mu < 4$

q $\left|\frac{(4-\mu)\vec{b} + \vec{a}}{4}\right| = |\vec{b} - \vec{a}|$

$(4 - \mu)^2 b^2 + a^2 = 4(b^2 + a^2)$

$3a^2 = ((4-\mu)^2 - 4)\ b^2$

$3(4-\mu) = (16 + \mu^2 - 8\mu - 4)$

$12 - 3\mu = \mu^1 - 8\mu + 12$

$\mu^2 - 5\mu = 0$

$\mu(\mu-5) = 0 \Rightarrow \mu = 0$

$\mu \neq 5$ as $\mu < 4$

(D) $$I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$$

$$I = \frac{4}{\pi}\int_{-\pi}^{\pi} \frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$$

$$I = \frac{4}{\pi}\int_{0}^{\pi} \frac{\cos\left(\frac{9x}{2}\right)}{\cos\left(\frac{x}{2}\right)} dx$$

$$2I = \frac{4}{\pi}\int_0^{\pi}\left(\frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}} + \frac{\cos\frac{9x}{2}}{\cos\frac{x}{2}}\right) dx = \frac{4}{\pi}\int_0^{\pi} \frac{\sin\left(\frac{9x}{2}+\frac{x}{2}\right)}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx = \frac{8}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x} dx$$

$$= \frac{8}{\pi}\int_0^{\pi} \frac{\sin 2x \cos 3x + \cos 2x \sin 3x}{\sin x} dx = \frac{8}{\pi}\int_0^{\pi} 2\cos x \cos 3x + (3 - 4\sin^2 x)\cos 2x \, dx$$

$$= \frac{8}{\pi}\int_0^{\pi} (\cos 4x + 3\cos 2x + \cos 3x - \cos x)\, dx = \frac{8}{\pi}\int_0^{\pi}\left[\frac{\sin 4x}{4} + \frac{3\sin 2x}{2} + \frac{\sin 3x}{3} - \sin x\right]_0^{\pi} = 0$$

38. Match the statements in **Column I** with those in **Column-II**

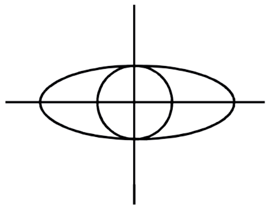
[**Note :** Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

Column I	**Column II**
(A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is contained in or equal to	(p) an ellipse with eccentricity $\frac{4}{5}$
(B) The set of points z satisfying $|z + 4| + |z - 4| = 10$ is contained in or equal to	(q) the set of points z satisfying Im z = 0
C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r) the set of points z satisfying $|\text{Im } z| \le 1$
(D) If $|w| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to	(s) the set of points z satisfying $|\text{Re } z| \le 2$
	(t) the set of points z satisfying $|z| \le 3$

Ans. (A) q, r (B) p (C) p,s,t (D) q,r,s,t

Sol.

(A) $\left|z - i|z|\right| = \left|z + i|z|\right|$



$(z - i|z|)\,(\bar{z} + i|z|) = (z + i|z|)(\bar{z} - i|z|)$

$z\bar{z} i\bar{z}|z| + z|z|i + |z|^2 = z\bar{z} + i\bar{z}|z| - iz|z| + |z|^2$

$2i\,(\bar{z}|z| - z|z|) = 0$

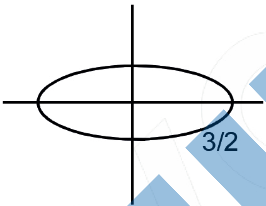
$\bar{z} = z|z| = 0$

$\therefore$ Im(z) = 0

(B) |z + 4| + |z – 4| = 10

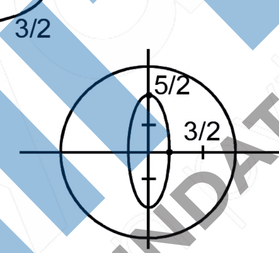
2ae = 8 2a = 10

e = $\frac{4}{5}$



a = 5

$b^2 = 25\left(1 - \frac{16}{25}\right)$



$b^2 = 9$

(C) $z = w - \frac{1}{\omega} = w - \frac{\bar{\omega}}{\omega\bar{\omega}} = \omega - \frac{\bar{\omega}}{4}$

$z = x + iy - \frac{x - iy}{4}$

$a + ib = \frac{3x}{4} + i\frac{5y}{4}$

$\left(\frac{4a}{3}\right)^3 + \left(\frac{4b}{5}\right)^2 = 4$

$\frac{x^2}{9} + \frac{4^2}{25} = \frac{1}{4}$

$e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

(D) $z = \omega + \frac{\omega\bar{\omega}}{\omega} = 2\text{Re}(\omega)$

$\therefore$ Im (z) = 0

PART C: PHYSICS

SECTION -I

Single Correct Choice Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

39. A tiny spherical oil drop carrying a net charge q is balanced in still aire with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m s^{-1}. Given g = 9.8 ms^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is

(A) 1.6×10^{-19}C (B) 3.2×10^{-19}C (C) 4.8×10^{-19}C (D*) 8.0×10^{-19}C

Sol. $2 \times 10^{-3} = \frac{2}{9}\frac{r^2 g \times p}{\eta}$

qE = mg = 6πηrv

$$q = \frac{6 \times \pi \times 1.8 \times 10^{-5} \times r \times 2 \times 10^{-3} \times 7}{8\pi \times 10^5}$$

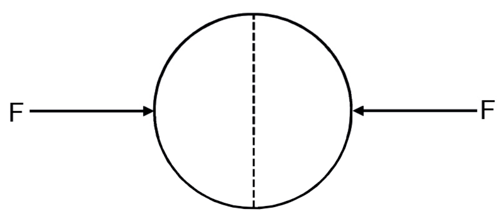
$$900 \times \frac{4}{3}\pi rg = 6\pi\eta rv$$

$$r^2 = \frac{6 \times 1.8 \times 10^{-5} \times 2 \times 10^{-3}}{1200 \times 9.8}$$

$$r = \frac{3}{7} \times 10^{-5}$$

$$q = \frac{2 \times 1.8 \times \frac{3}{7} \times 10^{-14} \times 2 \times 10^{-8} \times 7}{27 \times 10^5} = 8 \times 10^{-19}$$

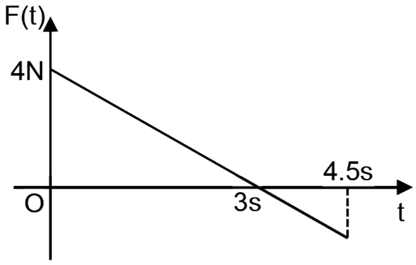
40. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (See figure). F is proportional to



(A*) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2 R$ (C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

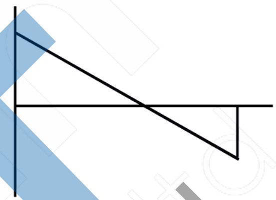
Sol. $p = \frac{\sigma^2}{2\epsilon_0} \Rightarrow F = \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$

41. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time-dependent force F(t) in the x direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(A) 4.50J (B) 7.50J (C*) 5.06J (D) 14.06J

Sol. Area = mv = $\frac{1}{2}\times4\times3-\frac{1}{2}\times2\times1.5$ = 4.5 = 2 × v



v = 2.25 m/s

$k=\frac{1}{2}\times2\times\left(\frac{4.5}{2}\right)^2=5.06\,J$

42. A biconvex lens of focal length 15cm is in front of a plane mirror. The distance between the lens and the mirror is 10cm. A small object is kept at a distance of 30 cm from the lens. The final image is

(A) virtual and at a distance of 16 cm from the mirror

(B*) real and at a distance of 16 cm from the mirror

(C) virtual and at a distance of 20 cm from the mirror

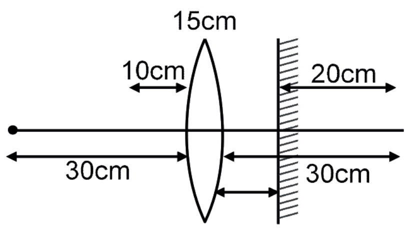
(D) real and at a distance of 20 cm from the mirror

Sol. u = + 10

$\frac{1}{v}-\frac{1}{10}=\frac{1}{15}$

$\frac{1}{v}=\frac{4}{30}+\frac{3}{30}$

v = 6 cm



43. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

(A) 0.02 mm (B) 0.05 mm (C) 0.1 mm (D*) 0.2 mm

Sol. L.C. = 1msd – 1 vsd = $\frac{4}{20}\text{msd}=0.2\text{mm}$

44. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50N and the speed of sound is 320 ms^{-1}, the mass of the string is

(A) 5 grams (B*) 10 grams (C) 20 grams (D) 40 grams

Sol. $$\frac{2}{2\times0.5}\times\sqrt{\frac{\frac{50}{m}}{0.5}}=\frac{1}{4\times0.8}\times320$$

$$\frac{2\times5}{\sqrt{m}}-100$$

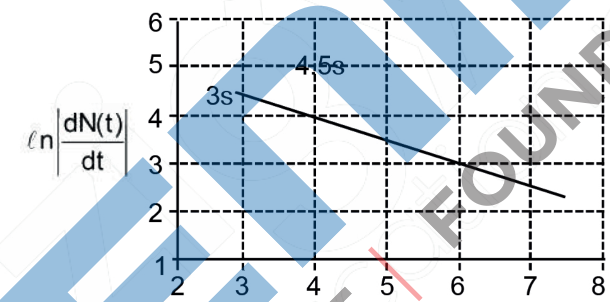
$$m=\frac{1}{100}=10\,gm$$

SECTION - II

Integer Type

This section contains 5 questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

45. To determine the half-life of the radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is



Ans. 8

Sol. $\frac{dN}{dt}=N_0e^{-\lambda t}$

$\ell n\left(\frac{dN}{dt}\right)=\ell nN_0-\lambda t$

$4=\ell n\,N_0$

$3=4-\lambda\times2$

$\lambda=0.5$ / year

$N=N_0e^{-\lambda t}$

$\frac{N}{N_0}=e^{-\lambda t}=e^{-2.08}=\frac{1}{e^{2.08}}=\frac{1}{8}$ = 8

46. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. In the initial temperature of gas is T_i (in Kelvin) and the final temperature is αT_i, the value of α is

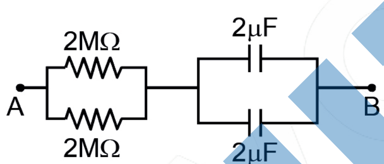
Ans. 4

Sol. $T_0 V_0^{7/5} = T\left(\frac{V_0}{32}\right)^{7/5} = T\left(\frac{V_0}{32}\right)^{2/5}$

$I = T_0 \times \left(32^{2/5}\right) = 4T_0$

47. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?

[Take : In 5 = 1.6, In 3 = 1.1]



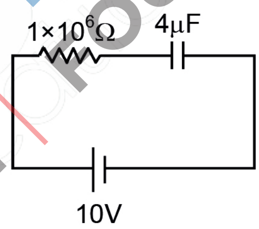
Ans. 2

Sol. $4 = Q = eV\,(1 - e^{-t/RC})$

$4 = 10$

$\frac{2}{5} = 1 - e^{-t/RC}$

$e^{-t/RC} = \frac{3}{5}$



$\frac{t}{RC} = 1.6 - 1.1 = 0.5$

$t = 1 \times 10^{-6} \times 4 \times 10^{-6} \times 0.5 = 2$ sec.

48. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour?

Ans. 3

Sol. $v = \frac{25}{3}$

$\frac{1}{v} + \frac{3}{25} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{3}{25} = \frac{25-30}{250} = \frac{-5}{250}$

$u_2 = -50$

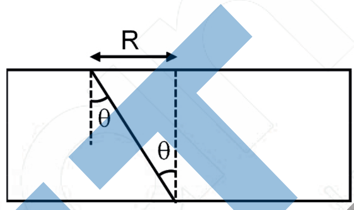
$$\frac{1}{v}+\frac{7}{50}=\frac{1}{10}=\frac{1}{v}=\frac{1}{10}-\frac{7}{50}=\frac{5-7}{50}=-25$$

vel. obj. = $\frac{25}{30}m/s=\frac{25}{30}\times\frac{18}{5}=3km/hr$

49. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

Ans. 6

Sol. $\sin\theta_c=\frac{1}{n}=\frac{3}{5}$



$\theta_c=37°$

$\frac{R}{8}=\tan\theta_c=\frac{3}{4}$

R = 6 cm

SECTION - III

Paragraph Type

This Section contains 2 paragraphs. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these questions had four choice (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 50 to 52

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

50. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ (C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D*) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

Sol. $I\omega=\frac{nh}{2\pi}$

$\frac{1}{2}I\omega^2=\frac{I^2\omega^2}{2I}=\frac{n^2h^2}{8\pi^2 I}$

51. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take h = 2 $\pi\times10^{-34}$ J s)

(A) 2.75×10^{-46} Kg m^2 (B*) 1.87×10^{-46} Kg m^2

(C) 4.67×10^{-47} Kg m^2 (D) 1.17×10^{-47} Kg m^2

Sol. $\frac{3\times h^2}{8\pi^2\lambda I}=hf=\frac{4}{\pi}\times10^{11}$

$I=\frac{3h}{8\pi\times4}\times10^{11}\times2\pi\times10^{-34}=\frac{3}{16}\times10^{-45}kgm^2$

52. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 am.u.), where 1 a.m.u. $=\frac{5}{3}\times10^{-27}$ Kg, is close to

(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m (C*) 1.3×10^{-10} m (D) 4.4×10^{-10} m

Sol. $I_{cm}=\mu x^2$

$\frac{3}{16}\times10^{-45}=\frac{12\times16}{12+16}\times\frac{5}{3}\times10^{-27}\times x^2=\frac{12\times16}{28}\times\frac{5}{3}\times10^{-27}\times x^2$

$x^2=\frac{3\times7}{256\times5}\times10^{-18}=\frac{21}{1280}\times10^{-18}$

$x=\frac{1}{8}\times10^{-9}m=1.25\times10^{-10}m$

Paragraph for Questions 53 to 55

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

53. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r << R) is

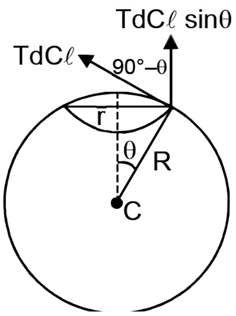
(A) 2 πrT (B) 2 πRT (C*) $\frac{2\pi r^2T}{R}$ (D) $\frac{2\pi R^2T}{r}$

Sol. TdCl sin θ

F_y = T sin $\theta\int dC\ell$

$=T\times\frac{r}{R}\times2\pi r$

$F_y=\frac{2\pi Tr^2}{R}$



54. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2} $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is aproximately

(A*) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

Sol. $R = \dfrac{2\pi r^2 T}{mg} \Rightarrow R = \dfrac{2\pi r^2 T}{\rho \times \frac{4}{3}\pi R^3 \times 9} \Rightarrow R^4 = \dfrac{3r^2T}{2\rho g} \Rightarrow R = (4)^{1/4} \times 10^{-3}$m

55. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B*) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

Sol. S.E. = $4\pi R^2 T = 4 \times 3.14 \times 1.96 \times 10^{-6} \times 0.11 = 2.7 \times 10^{-6}$ J

SECTION - IV

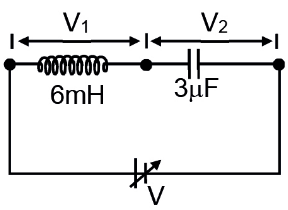
(Matrix Type)

This section contains 2 questions. Each questions has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s & t) in Column II. any given statement is Column I can have correct matching with one or more statement(s) given in Column II. For example, if for a given questions, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

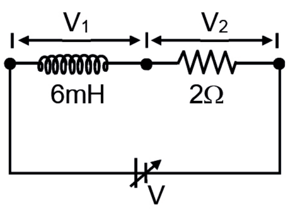
56. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz freqeuncy (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC rms for AC) flows through the circuit, then corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in Column I. Match th two

Column-I **Column-II**

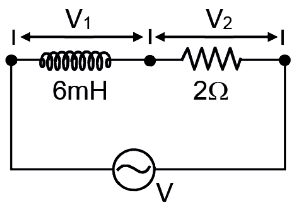
(A) $I \neq 0$, V_1 is proportional to I

(p) 

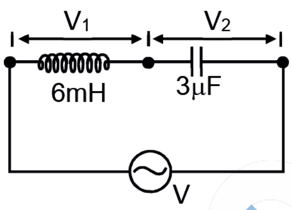
(B) $I \neq 0$, $V_2 > V_1$

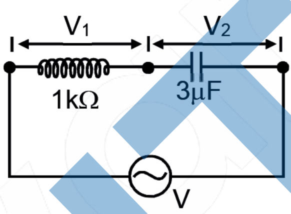
(q) 

(C) $V_1 = 0$, $V_2 = V$

(r) 

(D) $I \neq 0$, V_2 is proportional to I

(s) 

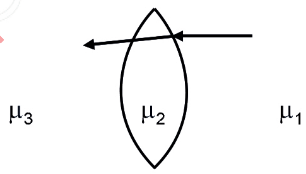
(t) 

Ans. **(A) r,s,t (B) q,r,s,t (C) p,q (D) q,r,s,t**

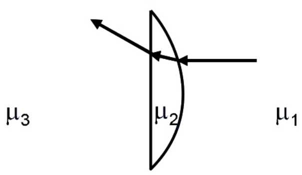
57. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped tan parent material refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2 are given. Match them to the ray diagrams shown in Column II.

Column-I **Column-II**

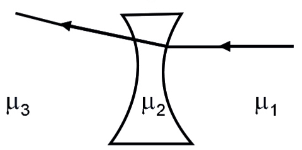
(A) $\mu_1 < \mu_2$

(p) 

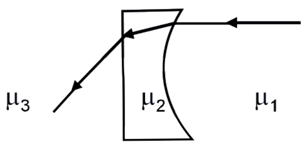
(B) $\mu_1 > \mu_2$

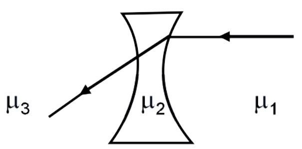
(q) 

(C) $\mu_2 = \mu_3$

(r) 

(D) $\mu_2 > \mu_3$

(s) 

(t) 

Ans. (A) p, r (B) q, s, t (C) p, r, t (D) q, s